

Open PhD Position at the Department of Mathematics “Tullio Levi-Civita”, University of Padova (A.Y. 2020-2021)

Dynamics and control of networked structures with Partial Differential Equations

Deadline for applications: June 16, 2020 (1pm).

Information: <https://www.unipd.it/en/node/1053>,
<https://dottorato.math.unipd.it/>

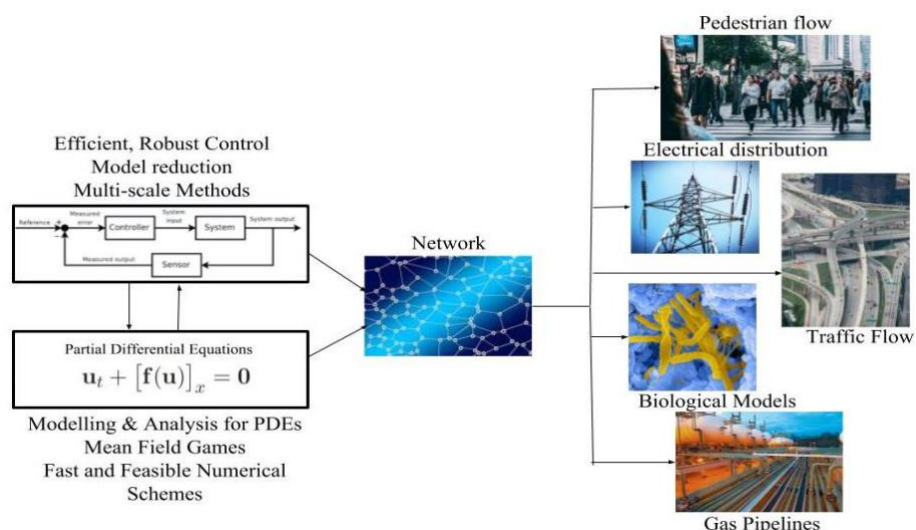
Description of the topic: The study of networks of interacting agents has produced a large number of mathematical models, with different formalisms and level of complexity. Such models try to catch some macroscopic features (e.g. self-organization) and eventually provide methods for simulation.

In relevant examples (crowds, social networks, bird flocks), networks are composed of hundreds or thousand of agents. This introduces a crucial difficulty, as standard analytical and numerical methods are useless.

A mathematically sound method to overcome such drawback is to describe the network via the density of agents, rather than the exact configuration of each of them. The resulting dynamics is a Partial Differential Equation, describing the time evolution of such density. The interaction between agents is well described by some non-local terms, e.g. by convolutions: as an example, the velocity of a pedestrian in the crowd is determined by its interaction with agents around himself.

The study of the interplay of macroscopic and microscopic (agent-based) models plays a key role also in the modelling of vehicular traffic. Microscopic follow-the-leader models take into account the dynamics of each single vehicles, whereas macroscopic ones deal with averaged quantities (density and velocity). The analysis of the convergence of deterministic particle schemes towards continuum PDEs has been recently addressed for the LWR (Lighthill-Whitham-Richards) model based on hyperbolic conservation laws.

Besides recent results about the analysis of such Partial Differential Equations, it is interesting to study the problem of controlling them. The idea is that an external regulator (e.g. a policy maker) can act on the network to achieve relevant goals: steer the network to a desired configuration, optimize a global cost, etc..



The goal of the thesis is twofold:

- Study the control of several relevant models of networks. A first example is the limit of the celebrated Cucker-Smale model, under an additional constraint of random lack of interaction. This is an interdisciplinary research, at the crossroad of mathematics and control engineering. It is the natural continuation of researches by F. Rossi in this field, that were published in one of the top engineering journals (*Automatica*).
- A second example concerns the control of the many particle limit of discrete (ODE-based) models of traffic flow on a network.
Provide general strategies for ensuring controllability and compute optimal control of such equations, for example to address management problems of vehicular traffic or of pedestrian flow in densely crowded regions by means of "sparse" controls acting on the system (autonomous vehicles, toll gates, cellular phones).

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Research Group: The project is strengthened by well-established international collaborations of the advisors with leading experts of the research fields of the topic. The recruited student will spend 3 to 6 months in one (or two) of the following international institutions:

- Université Paris Dauphine PSL - Ceremade, <https://www.ceremade.dauphine.fr/>
 - University of Oxford - Mathematical Institute, <https://www.maths.ox.ac.uk/>
 - University of Rutgers Camden - Department of Mathematical Sciences, <https://math.camden.rutgers.edu/>
 - North Carolina State University - Department of Mathematics, <https://math.sciences.ncsu.edu/>
- and he will benefit of meetings and scientific networking activities in which the advisors are involved.

Admission Requirements: Highly motivated candidates are expected to have excellent analytical reasoning and a solid mathematical background, in particular to be familiar with basic notions of Ordinary Differential Equation and Partial Differential Equations, as well as to share an interest for applied mathematics investigations.

References

- F. Ancona, A. Cesaroni, G.M. Coclite, M. Garavello, On the optimization of conservation law models at a junction with inflow and outflow distribution controls. *SIAM J. Control Optim.* 56 (2018), no. 5, 3370-3403.
- A. Bressan, K. T. Nguyen, Conservation law models for traffic flow on a network of roads. *Network and Heterogeneous Media* 10 (2015), no. 2, 255-293.
- M. Caponigro, B. Piccoli, F. Rossi, E. Trelat, Sparse Jurdjevic-Quinn stabilization of dissipative systems, *Automatica*, 86, pp. 110-120, 2017.
- P. Cardaliguet, N. Forcadel, From heterogeneous microscopic traffic flow models to macroscopic models, 2019, arXiv:1907.02310.
- M.L. Delle Monache, B. Piccoli, F. Rossi, Traffic regulation via controlled speed limit, *SIAM Journal on Control and Optimization*, 55(5), pp. 2936-2958, 2017.
- M. Di Francesco and M. D. Rosini. Rigorous derivation of nonlinear scalar conservation laws from Follow-the-Leader type models via many particle limit. *Archive for Rational Mech. and Analysis*, 217(3):831-871, 2015.
- P. Goatin, F. Rossi, A traffic flow model with non-smooth metric interaction: well-posedness and micro-macro limit, *Comm. Math. Sciences*, vol. 15 (1), pp. 261-287, 2017.
- M. Lighthill, G. Whitham, On kinematic waves. II. A theory of traffic flow on long crowded roads, *Proceedings of the Royal Society of London: Series A*, 229 (1955), 317-345.
- B. Piccoli, F. Rossi, E. Trelat, Control to flocking of the kinetic Cucker-Smale model, *SIAM J. Mathematical Analysis* 47, no. 6, pp. 4685-4719, 2015.
- P. I. Richards, Shock waves on the highway, *Oper. Res.*, 4 (1956), 4251.
- M. D. Rosini. Macroscopic models for vehicular flows and crowd dynamics: theory and applications. *Understanding Complex Systems*. Springer, Heidelberg, 2013.