Doctoral Program in Mathematical Sciences
Department of Mathematics “Tullio Levi-Civita”
University of Padova

Doctoral Program in Mathematical Sciences

Catalogue of the courses 2025-2026

Updated May 27th, 2025

INTRODUCTION

This Catalogue contains the list of activities offered to the Graduate Students in Mathematical Sciences for the year 2025-2026.

The activities in this Catalogue are of three types.

1. Courses offered by the Graduate School (= Courses of the Doctoral Program)
2. Courses offered by one of its curricula.
3. Other activities:
   a) selected courses offered by the PhD school in Information Engineering;
   b) selected courses offered by other PhD schools or other Institutions;
   c) reading courses

(This offer includes courses taught by internationally recognized external researchers. Since these courses might not be offered again in the near future, we emphasize the importance for all graduate students to attend them.)

Taking a course from the Catalogue gives an automatic acquisition of credits, while crediting of courses not included in the Catalogue (such as courses offered by the Scuola Galileiana di Studi Superiori, Summer or Winter schools, Series of lectures devoted to young researchers, courses offered by other PhD Schools) is possible but it is subject to the approval of the Executive Board. **Moreover, at most one course of this type may be credited.**

We underline the importance for all students to follow courses, with the goal of **broadening their culture in Mathematics**, as well as developing their knowledge in their own area of interest.

REQUIREMENTS FOR GRADUATE STUDENTS

Within the **first two years of enrollment** all students are required to

- pass the exam of at least four courses from the catalogue, among which at least two must be taken from the list of “Courses of the Doctoral Program”, while at most one can be taken among the list of “reading courses”
- participate in at least one activity among the “soft skills”
- attend at least two more courses (for such activities the PhD student must produce a brief summary on what she/he learned. These summaries should be attached to the annual report)

In the case of a research period abroad lasting at least one semester during the first or second year, a PhD student may request the Faculty Board to replace certain courses (excluding the Program Courses) with other courses offered at the foreign institution during their stay. The number of additional replacements should be proportional to the length of the research period, and the total commitment should be equivalent to the normal commitment. For example, if a PhD student spends a full semester abroad, they may request to replace two of the “other courses,” which is one more than what is allowed for students who do not spend extended research periods abroad. Each interested student is expected to submit all their requests for the approval of courses not included in the catalogue at the same time.

Students are warmly encouraged to take more courses than the minimum required by these rules, and to commit themselves to follow regularly these courses. It is also recommended that one half of the exams are taken during the first year. At the end of each course the instructor will inform the Coordinator and the Secretary on the activities of the course and of the registered students.

Students **must register** to all courses of the Graduate School that they want to attend, independently of their intention to take the exam or not. We recommend to register as early as possible: the Graduate School may cancel a course if the number of registered students is too low. If necessary, the registration to a Course may be canceled.

Courses attended in other Institutions and not included in the catalogue.

Students activities within Summer or Winter schools, series of lectures devoted to young researchers, courses offered by the Scuola Galileiana di Studi Superiori, by other PhD Schools or by PhD Programs of other Universities may also be credited, according to whether an exam is passed or not; the student must apply to the Coordinator and crediting is subject to approval by the supervisor and the Executive board. We recall that **at most one course** not included in the Catalogue may be credited.

Seminars

a) All students, during the three years of the program, must attend the **Colloquia of the Department** and participate regularly in the Graduate Seminar ("**Seminario Dottorato**"), whithin which they are also required to deliver a talk and write an abstract.
b) Students are also strongly encouraged to attend the seminars of the research groups that are relevant for their work.

HOW TO REGISTER AND UNREGISTER TO COURSES

The registration to a Course must be done online.
Students can access the **online registration form** in the dedicated page of the Doctoral Course website at https://dottorato.math.unipd.it/current-activity/FutureActivities clicking on "click to enroll" of the chosen courses. The registration lists can be reached also via the website of the Department of Mathematics at https://prev-www.math.unipd.it/userlist/
In order to register, fill the registration form with all required data, and validate with the command "Subscribe". The system will send a confirmation email message to the address indicated in the registration form; please save this message, as it will be needed in case of cancellation.

Registration to a course implies the commitment to follow the course.

Requests of **cancellation** to a course must be submitted in a timely manner, and **at least one month before the course** (except for courses that begin in October and November) using the link indicated in the confirmation email message.

REQUIREMENTS FOR PARTICIPANTS NOT ENROLLED IN THE GRADUATE SCHOOL OF MATHEMATICS

The courses in this catalogue, although part of activities of the Graduate School in Mathematics, are open to all students, graduate students, researchers of this and other Universities.
For organization reasons, external participants are required to **communicate their intention** (phd.math@math.unipd.it) to take a course at least two months before its starting date if the course is scheduled in January 2026 or later, and as soon as possible for courses that take place until December 2025.
In order to **register,** follow the procedure described in the preceding section.
Possible **cancellation** to courses must also be notified.

List of Courses

Courses of the Doctoral Program

1. Prof.ssa Alessandra Bianchi
   Random Graphs and Networks **DP-1,2**

2. Prof.ssa Laura Caravenna
   Topics in Calculus of Variations **DP-3,4**

3. Prof. Luis C. Garcia-Naranjo
   Lie Groups and Symmetry **DP-5**

4. Proff. Giulio Peruginelli, Andriy Regeta
   Reflection Groups **DP-6,7**

Courses of the "Mathematics" area

1. Prof. Oren Ben-Bassat
   Principal Bundles **M-1**

2. Prof.ssa Giovanna Carnovale
   The Drinfeld double of a finite group **M-2,3**

3. Proff. Francesco Fassò, Nicola Sansonetto
   Nonholonomic Mechanical Systems **M-4,5**

4. Dott. Mattia Fogagnolo, Prof.ssa Valentina Franceschi
   Geometric aspects of PDEs **M-6,7**

5. Proff. Daniel Labardini-Fragoso, Alberto Tonolo, Jorge Vitoria
   The Maximal Graphs, algebras and representations **M-8,9**

6. Prof. Jakob Scholbach
   Intersection Theory **M-10**

Courses of the "Computational Mathematics" area

1. Dott.ssa Ofelia Bonesini
   Volterra Equations and their Markovian Lift(S) **MC-1**

2. Dott. Alekos Cecchin
   Stochastic Optimal Control **MC-**

3. Dott. Federico Nudo
   General enrichment strategies for finite element methods to solve Poisson problem with Dirichlet boundary conditions **M-**

4. Prof. Gilles Pagès
   Functional convex ordering of stochastic processes: a constructive approach with applications to Finance **MC-**

5. Dott. Federico Piazzon
   Linear Parabolic Equations in Hilbert Spaces: analysis and numerical approximation **MC-**

Courses of the Doctoral Program

Random Graphs and Networks

Alessandra Bianchi[1]

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Email: alessandra.bianchi@unipd.it

Timetable: 24 hrs. ; February 2026

ECTS: 4

SS: MATH-03/B

Introduction and background requirements: Basic knowledge of probability theory: discrete random variables, finite and countable probability spaces, convergence theorems (law of large number, central limit theorem).

Aim: Random graph theory sits at the intersection of probability, combinatorics, and graph theory, offering elegant and rigorous methods to study complex networks. These methods provide insights into both real-world phenomena and abstract mathematical structures.
The aim of this mini-course is to equip PhD students with a rigorous understanding of both the theoretical and applied aspects of random graphs as models for complex networks. Upon completion, students are expected:

- to achieve a solid understanding of fundamental concepts in random graph theory, including classical random graph models and properties relevant to real-world networks;
- to be able to implement the main techniques involved in the study of random graphs, including probabilistic methods, combinatorial tools, and analytical methods;
- to gain insights into advanced topics, such as problems in community detection or characterizing the spectrum of random graphs, and to develop the skills necessary to understand scientific papers on these subjects.

Examination and grading: Seminar on a paper.

Course contents: Complex networks have captured the attention of the scientific community in recent years due to their prevalence in a wide variety of real-world scenarios, such as social networks, biological systems, and technological infrastructures. These networks exhibit large-scale behaviors that reveal common properties, notably the "small-world" effect and the "scale-free" phenomenon. Random graphs serve as mathematical models that facilitate the analysis of these large-scale features. Roughly, random graphs can be described as random variables taking values on a set of graphs, hence well suited to capture both probabilistic and combinatorial aspects of the real-world networks.

The course will focus on different classes of random graphs. We will start from the definition of the Erdos-Rényi random graph, one of simplest model one could think of. Despite its simplicity, this model presents relevant and unforeseen large-scale features that will be discussed along the course, including an interesting phase transition related to presence of a giant connected component. Keeping in mind the properties of real networks, we will then introduce and discuss three different families of random graphs: The Inhomogeneous Random Graph, the Configuration Model and the Preferential Attachment Model. Each model captures different features of real-world networks, such as heavy-tailed degree distributions and small world behavior, while maintaining mathematical tractability. The course will conclude discussing more applied topics such as generative models for community detection in networks. In particular, the Stochastic Block Model (SBM) and its connection with belief propagation and the reconstruction problem on trees will be presented as time permits.

Lectures (tentative schedule)

1. Basic setting: graphs, trees, random graph setting, and main properties of the real-world networks.
2. Erdös-Rényi (ER) random graphs: Uniform and Binomial model; monotonicity and thresholds.
3. ER random graphs structure: trees containment, Poisson paradigm, largest component, connectivity.
4. Exploration process of a graph and a random walk perspective. Tool: Branching Processes.
5. Emergence of a giant component in ER- random graphs.
6. Inhomogeneous random graphs (IRG): degree sequence and scale-free property.
7. Configuration Model (CM): construction and simplicity probability. Uniform random graphs.
8. Phase transition and small world phenomenon in the IRG and in the CM. Tool: Multi-type branching process.
9. Preferential Attachment Model (PAM): construction, scale free and small world properties.
10. Perspectives I: Community structure and community detection.
11. Perspectives II: Stochastic Block Model (SBM) and reconstruction on trees.
12. Problem for solution: the spectrum of random graphs.

References:

1. R. van der Hofstad. Random graphs and complex networks. Vol. 1. Cambridge Series in Statistical and Probabilistic Mathematics, [43]. Cambridge University Press, Cambridge, 2017. (available on the author webpage)
2. R. van der Hofstad. Random graphs and complex networks. Vol. 2. Cambridge Series in Statistical and Probabilistic Mathematics, [54]. Cambridge University Press, Cambridge, 2024. (available on the author webpage)
3. A. Frieze, M. Karoński. Introduction to random graphs. Cambridge University Press, Cambridge, 2016. (available on the author webpage)
4. R. van der Hofstad. Stochastic processes on random graphs. Lecture notes for the 47th Summer School in Probability Saint-Flour, 2017. (available on the author webpage)
5. E. Abbe. Community Detection and Stochastic Block Models: Recent Developments. Journal of Machine Learning Research, 18(177), 86 pp, 2017.

Topics in the Calculus of Variations

Laura Caravenna[1]

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Timetable: 24 hrs. A weekly 2-hours lecture for 12 weeks starting from November 2025. Precise dates will be on the Calendar of Activities at https://dottorato.math.unipd.it/calendar/,Torre Archimede, Room 2BC30

ECTS: 4

SS: MATH-03/A

Course requirements: Real analysis, some notions of basic PDEs and some functional analysis are welcome, for instance, chapters 1, 3, 4, 8, and 9 of Brezis' book on functional analysis. The main required tools will be recalled during the course.

Examination and grading: Oral exam, based either on a problem set or on a research paper

Aim: With the first part of the course students will learn the fundational aspects in the Calculus of Variations. The second part will allow them to master more specialized tools from the branch of optimal transport in their applications, also to some evolution PDEs.

Course contents: The calculus of variations is a cornerstone of mathematical analysis and optimization, focusing on finding functions that minimize or maximize certain quantities, typically expressed as integrals. This theory underpins much of modern physics and engineering, providing the mathematical framework for principles like the least action in mechanics and the optimal shapes and configurations in structural design. It has broad applications, from determining geodesics and surfaces of minimal area to optimizing control systems and studying the behavior of complex systems described by partial differential equations. In addition to its classical uses, calculus of variations has fueled advances in fields like materials science, image processing, and machine learning, where problems often require identifying optimal configurations or shapes.

Optimal transport theory is a relatively new branch of the calculus of variations. The original question involves finding the most efficient ways to move distributions of mass or probability from one configuration to another, while minimizing a given cost function. Initially formulated by Gaspard Monge in 1781 and later generalized by Leonid Kantorovich in 1940-42, it has become a powerful tool both for pure mathematics and for applied problems of redistribution and matching in economics, physics, data science, and beyond.

The first part of the course covers foundational aspects. As a warm-up, I will introduce one-dimensional variational problems and optimality conditions like Euler-Lagrange equations through classical examples, such as the geodesic problem. I will then discuss Monge's formulation of Optimal Transport Problems and its limitations, leading to Kantorovich's relaxation of the problem: here the existence of minimizers directly follows from the direct method of the calculus of variations. I will introduce Kantorovich-Rubinstein duality, and necessary and sufficient conditions for optimality. There will be a particular focus on c-cyclical monotonicity, relating it to classical concepts in convex analysis that it generalizes. I will then discuss the problem of existence of optimal maps with a special focus on Brenier's theorem for the quadratic cost function. After this introductory part, I will select applications of optimal transport, also depending on the interest of the audience. Possible choices include: connection with the Monge-Ampère equation, Wasserstein distances and their properties, curves in Wasserstein spaces and their relation to the continuity equation, geodesics, Benamou-Brenier formula, characterization of AC curves inWasserstein spaces, introduction to gradient flows in metric spaces and to the JKO minimization scheme for some evolution equations, price equilibria in economic models.

Bibliography:

Notes from lectures will be available. Relevant books for consultation are

- A. Figalli, F. Glaudo: An Invitation to Optimal Transport, Wasserstein Distances and Gradient Flows, 2022
- L. Ambrosio, E. Brué and D. Semola: Lectures on Optimal Transport, Springer, 2022
- F. Santambrogio: Euclidean, Metric, and Wasserstein Gradient Flows: an overview, Bulletin of Mathematical Sciences, available online, 2017
- F. Santambrogio: Optimal Transport for Applied Mathematicians, Birkhauser, 2015
- L. Ambrosio, N. Gigli: A User's Guide to Optimal Transport, 2012
- L. Ambrosio, N. Gigli, G. Savaré: Gradient Flows in Metric Spaces and in the Space of Probability Measures, Birkhauser, 2005
- C. Villani: Topics in Optimal Transportation, American Mathematical Society, 2003

Lie Groups and Symmetry

Prof. Luis C. García-Naranjo[1]

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Timetable: 24 hrs. Precise schedule of the lectures to be defined.

ECTS: 4

SS: MATH-04/A

Introduction and background requirements: basic knowledge of differential geometry. The course is addressed to all students.

Aim: The course aims at providing an introduction to the theory of Lie groups and their actions, which is a topic of broad interest in several areas of Mathematics and applications. After covering the fundamentals of the subject, the course will provide some examples of use of Lie groups in the study of ODEs with symmetry.

Examination and grading: oral examination on the topics covered during the course.

Course contents: Lie groups and their differential and group structures (left and right trivializations, Lie algebra of a Lie group, homomorphisms, exponential map, (co)adjoint action), structure of compact Lie groups (maximal tori, Weyl chambers); classical matrix groups and their properties; relationship between Lie groups and Lie algebras, Lie's Theorems and the Baker-Campbell-Hausdorff formula; differentiable actions of Lie groups on manifolds, quotient spaces (for proper actions), Palais slice theorem, invariant vector fields; reduction of invariant vector fields; applications to ODEs with symmetry (relative equilbria and relative periodic orbits, connection with Floquet theory, reduction and reconstruction, integrability).

References:

1. J.J. Duistermaat, J.A.C. Kolk, Lie Groups. (Springer, 2000).
2. A. Baker, Matrix groups. An introduction to Lie group theory. (Springer, 2002)
3. J. Lee, Introduction to Smooth manifolds. 2nd edition. (Springer, 2013)
4. R. Cushman, J.J. Duistermaat and J. Śnyaticki, Geometry of Nonholonomically Constrained Systems. (World Scientific, 2010).

Reflection groups

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Timetable: 24 hrs.; expected starting date TBA

ECTS: 4

SS: MATH-02/A

Introduction and background requirements: Symmetry is a crucial concept in mathematics and the first natural examples of symmetries one may think of are reflections. Groups generated by reflections (reflection groups) include well-known families, such as the symmetric groups and the dihedral groups. Reflection groups have very special properties, that can be seen for instance in their group structure, and in the nature of their orbit spaces. Classical corner stones in the theory are the classification of finite reflection groups in a Euclidean space by means of Coxeter graphs -including the classification of Platonic solids- and the Chevalley-Shephard-Todd's theorem. The latter characterizes finite groups generated by complex reflections acting on a linear space as those for which the ring of invariant polynomial functions is a ring of polynomials (i.e., the orbit space is again linear). After getting a grip on these basic facts, the course is intended to move on to the role that reflection groups play in singularity theory.

Aim: (Learning goals - Intended Learning Outcomes) The aim of the course is to stress the special nature of groups generated by reflections and the role of symmetry in different situations in mathematics.

Expected knowledge, abilities and competences: Basic notions on groups, rings and vector spaces that are usually covered in a bachelor degree in mathematics. If needed, definitions and basic theorems would be recalled along the course.

Examination and grading: Solving exercises and/or giving a seminar on a paper related to the content of the course.

Course contents:

- Reflections in a Euclidean space and complex reflections. Reflection groups. Examples.
- Coxeter groups and Coxeter graphs. The classification of finite reflection groups. Platonic solids.
- Crystallographic groups (Weyl groups).
- How do we parametrize orbits? Basics on commutative algebra and invariant theory. Orbit spaces and fundamental regions.

- Invariant polynomials. The case of the symmetric group.
- Chevalley-Shephard-Todd theorem with examples.
- Degrees of a finite reflection group.

(if time permits)

- Basics on affine algebraic varieties.
- Platonic solids, finite subgroups of $\mathrm{SL}_2(\mathbb{C})$ and Kleininan singularities.

References:

[B] N. Bourbaki, Elements of Mathematics, Chapter IV, Coxeter Groups and Tits systems and Chapter V, Groups Generated by Reflections, Springer, English translation by Andrew Pressley, from the 1968 original version.

[H] J. Humphreys, Reflection Groups and Coxeter Groups, Cambridge University Press, 1992.

[R] A. Regeta, Lectures on Reflection Groups and Invariant Theory, lecture notes, available at https://andriyregeta.wixsite.com/homepage

[S] P. Slodowy, Platonic solids, Kleinian singularities, and Lie groups, in: Proceedings of the Third Midwest Algebraic Geometry Conference held at the University of Michigan, Ann Arbor, USA, November 14-15, 1981 Ed; I Dolgachev.

[D] I. Dolgachev, Reflection groups in algebraic geometry, Bull. A.M.S. 45 (2008), 1–60.

Courses of the “Mathematics” area

Principal Bundles

Oren Ben-Bassat[1]

[1] *Department of Mathematics, University of Haifa, Israel*
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Timetable: 16 hrs. First lecture on October-November 2025, Torre Archimede, Room 2BC30

CFU/ECTS: 3

SSD: MATH-02/B

Course requirements: It will be helpful to come with some familiarity with topics like commutative algebra, homological algebra, vector bundles, differential geometry, algebraic geometry, different types of cohomology. We will try to fill in the necessary category theory as we go along.

Examination and grading: Oral presentation of an argument related to the topics presented during the lectures.

Aim: The course provides an introduction to the theory of principal bundles and shows some applications in different fields of mathematics, ranging from arithmetic to mathematical physics.

Course contents:

1. Short introduction to Grothendieck topologies and sheaves. E' tale cohomology, Cěch cohomology. Examples.
2. Principal bundles and torsors in topology, arithmetic geometry, complex analytic geometry, differential geometry, and algebraic geometry.
3. Groupoids, moduli spaces of vector bundles, vector bundles on the projective line and other algebraic curves.
4. Stable bundles, Higgs bundles, Hitchin systems and their quantization.
5. Topological Quantum Field Theories and Frobenius algebras.
6. Defining Topological Quantum Field Theories with G-bundles.

Optional topics:

- Related topics in representation theory and group cohomology

Bibliography:

- Hitchin systems and their quantization, Pavel Etingof, Henry Liu, https://arxiv.org/abs/2409.09505
- "Frobenius algebras and 2D topological quantum field theories" (short version), Joachim Kock https://mat.uab.cat/ kock/TQFT/FS.pdf
- "Vector Bundles and K-theory", by Allen Hatcher https://pi.math.cornell.edu/ hatcher/VBKT/VB.pdf

The Drinfeld double of a finite group

Giovanna Carnovale[1]

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Timetable: 16 hrs. ; expected starting date: presumably january 2026

CFU/ECTS credits: 3

SS: MATH-03/A

Introduction and background requirements: In his 1986 Fields Medal paper [6], Drinfeld introduced the notion of a ring with special features nowadays called the Drinfeld double, one of the main goals being the production of solutions of the quantum Yang-Baxter equation from statistical mechanics. The construction is inspired by a similar construction on Poisson-Lie groups and requires an initial datum given by a general Hopf algebra. The case in which the starting datum is a finite group G is already extremely rich: the double $D(G)$ occurs in the work of Dijkgraaf, E. Verlinde, H. Verlinde and Vafa [5] in conformal field theory, in Lusztig's work on representations of finite groups of Lie type [8], in the study of mapping class groups of surfaces and knot and link invariants, in Verlinde's method to count morphisms from fundamental groups of surfaces to a given group G, [9], in Andruskiewitsch and Schneider's program for the classification of Hopf algebras [1], in Cibils and Rosso's classification of path algebras with a Hopf algebra structure, [4]. It is usually studied through its representations, that is, through the different ways in which we can see the elements of $D(G)$ as endomorphisms of a given vector space. Representations for $D(G)$ can be interpreted in different ways: for example their geometric interpretation in terms of vector bundles lead to the non-abelian Fourier transform in [8]. Verlinde provided a striking formula of its fusion rules (decomposition of tensor products of representations) in terms of group theoretical data making use of conformal field theory only [9]: an algebraic proof of this formula can be given in terms of Fourier transforms on $G \times G$, [7]. Several further applications and interpretations of the representations of $D(G)$ are listed in the survey [3].

The course will review some of the key features of the Drinfeld double of a finite group, its representations, and their applications in topology, developing the theory from scratch and relying for a big part on the basic treatment in [2]. Prerequisites are: basic notions of linear algebra (including the tensor product of vector spaces) and algebra covered in a standard bachelor in mathematics. No prior knowledge of Hopf algebras or representation theory are required. Hopf algebraic tecnichalities will be kept to a minimum.

Aim: (Learning goals - Intended Learning Outcomes)
The goal of the course is to offer a glimpse of the Drinfeld double of a finite group and some of its applications in representation theory and topology. It should serve as a tool to see how changing point of view on the same mathematical object can lead to unexpected results.
Expected knowledge, abilities and competences

We expect that through the rich example of $D(G)$, the participants will acquire familiarity with standard ideas from Hopf algebra theory, such as representations, tensor products, braidings, their potential applications in topology.

Examination and grading: Solutions of some exercises during the course, followed by an oral discussion.

Course contents:

- Basic notions on representations and characters;
- Hopf algebras and tensor products of representations. Quasitriangular Hopf algebras and the quantum Yang-Baxter equation;
- The different realizations of the Drinfeld double $D(G)$ of a group G;
- Different realizations of the representations of $D(G)$;
- The braid group; knot and link invariants from representations of $D(G)$;
- Mapping class groups: the case of the torus, and representations of $\mathbf{S}L_2(\mathbb{Z})$ obtained from $D(G)$;
- Fourier transforms for G and $D(G)$; Verlinde formula for the decomposition of the tensor product of representations of $D(G)$
- Cibils and Rosso's classification of path algebras with a Hopf algebra algebra structure.

References

[1] N. Andruksiewitsch, H.J. Schneider, On the classification of finite-dimensional pointed Hopf algebras, Ann. Math. 171(1), (2010), 375–417.

[2] M. Broué, From Rings and Modules to Hopf Algebras. One Flew Over the Algebraist's Nest, Springer Nature Switzerland, (2024).

[3] G. Carnovale, N. Ciccoli, E. Collacciani, The versatility of the Drinfeld double of a finite group, survey, Arxiv:2410.11978.

[4] C. Cibils, M. Rosso, Algèbres des chemins quantiques, Adv. Math. 125, 171–199 (1997).

[5] R. Dijkgraaf, C. Vafa, E. Verlinde, H. Verlinde, The operator algebra of orbifold models, Comm. Math. Phys. 123(3), 485–526, (1989).

[6] V.G. Drinfel'd, Quantum groups, in: Proceedings of the I.C.M., Berkeley, (1986), American Math. Soc., 1987, 798?820.

[7] T.H. Koornwinder, B. J. Schroers, J. K. Slingerland, F. Bais, Fourier transform and the Verlinde formula for the quantum double of a finite group, Journal of Physics A Mathematical and General 32(48),8539–8549, (1999).

[8] G. Lusztig, Characters of Reductive Groups over a Finite Field, Princeton University Press (1984).

[9] G. Mason, The quantum double of a finite group and its role in conformal field theory. In: Groups '93 Galway/St. Andrews, 2, 405–417. London Math. Soc. Lecture Note Ser., 212, Cambridge University Press, Cambridge, (1995).

Nonholonomic Mechanical Systems

Francesco Fassó[1], Nicola Sansonetto[2]

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Timetable: 16 hrs., Room 2BC30, Torre Archimede.

CFU/ECTS: 3

SS: MATH-04/A

Introduction and background requirements: In mechanics, a nonholonomic constraint is a restriction on the possible velocities of a holonomic system without restricting its possible configurations. Examples are systems of rigid bodies whose contact points are constrained to have a preassigned velocity and the possibility to manoeuvre a car into parallel parking despite the fact that the wheels cannot move transversally. Nonholonomic mechanics is very different from Lagrangian mechanis because the equations of motion do not come from a variational principle. Many properties of the dynamics are still not completely understood and the subject is an active field of research, which is of interest in various areas of Mathematics and Engeneering. The aim of the course is to give an introduction to the subject, focussing mostly on the dynamical aspects and presenting the students to recent progress and open questions in the field. Prerequisites are basic notions of classical mechanics (Lagrangian mechanics), of the qualitative theory of ODEs and of analysis and differential geometry. More advanced topics (e.g., Frobenius theorem, Lie groups actions) will be reviewed.

Examination and grading: Verification of the comprehension and knowledge of the core topics covered in the course through a colloquium and/or the presentation of specific topics studied individually.

Aim: The aim of the course is to provide a basic, though solid, knowledge of the geometric structure and of the dynamics of nonholonomic mechanical systems, leading the students to the level of understanding the current literature.

Course contents: Nonholonomic constraints. Distributions and Frobenius theorem. Ideal constraints and D'Alembert principle. The equations of motion for linear and nonlinear constraints. The role and structure of the reaction forces. Basic examples of nonholonomic mechanical systems. First integrals: energy, momenta, the nonholonomic Noether theorem. Based on the interest of the partecipants and on the time availability, one or more of the following topics might be covered: (non)existence of conserved measures; reduction under symmetry; integrability; control and trajectory planning of nonholonomic systems.

References:

1. Lecture notes will be prepared and made accessible to the students during the course.

2. L. García-Naranjo, Geometry and dynamics of nonholonomic systems. Lecture Notes for a course given in this Doctoral School in 2021.
3. Ju. I. Neĭmark and N.A. Fufaev, Dynamics of Nonholonomic Systems (AMS, 1972).
4. F. Bullo and A.D. Lewis, Geometric Control of Mechanical Systems (Springer, 2004).
5. R. Cushman, J.J. Duistermaat and J. Snyaticki, Geometry of Nonholonomically Constrained Systems (World Scientific, 2010).
6. A. Bloch, Nonholonomic Mechanics and Control (Springer, 2015).

Topics in the Calculus of Variations

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Timetable: 16 hrs. Spring 2026,Torre Archimede, Room 2BC30

CFU/ECTS: 3

SS: MATH-03/A

Course requirements: basics in PDEs, calculus of variations, geometry of submanifolds.

Examination and grading: seminar about a research paper in the topic.

Aim: The course aims at providing the attendees some deep connections among fundamental properties of elliptic PDEs and prototypical problems in geometric analysis.

Course contents: In this course we deal with some fundamental properties of basic elliptic PDEs, and build on them to discuss powerful results in the geometric analysis and regularity theory of submanifolds. The techniques and the presentation are suited to be applied in more general geometries and to more general equations. After recalling some basics on the geometry of submanifolds and the regularity theory for elliptic pdes, the topics treated will include:

- Sharp gradient bounds for solutions to Laplace equations, leading in turn to the characterization of spheres as the only closed surfaces with constant mean curvature (Alexandrov theorem). This result and its proof will then be compared with Serrin's overdetermined problem.
- Regularity of solutions to elliptic PDEs, minimal graphs and regularity of sets with bounded mean curvature. Monotonicity formula for minimal surfaces and Allard's Theorem.
- Almgren's frequency function and estimates of the critical set of harmonic functions, in connection with the monotonicity formula for stationary submanifolds.

Bibliography:

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- Y. TONEGAWA Brakke's Mean Curvature Flow An Introduction Springer, 2019.
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- B. WHITE A local regularity theorem for mean curvature flow, Ann. of Math., 2005.

Graphs, algebras and representations

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Timetable: 16 hrs., Room 2BC30, Torre Archimede.

CFU/ECTS: 3

SS: MATH-02/A

Introduction and background requirements: A directed graph, or quiver, is a set of vertices and a set of arrows between those vertices. An example of a quiver is

$$\mathcal{A}_3 := 1 \longrightarrow^{\alpha} 2 \longrightarrow^{\beta} 3$$

A path in the graph is a sequence of composable arrows starting in a vertex and ending in a vertex. In the quiver above, the set of paths is e_1, e_2, e_3, $\alpha, \beta, \beta\alpha$, where e_1 denotes the path starting and ending in vertex i (said to be a lazy path) with no arrow in between, and the path $\beta\alpha$ denotes the composition of β with α, starting in vertex i and ending in vertex 3. If we fixa field $\mathbb{K}$, we may consider the vector space $\mathbb{K}Q$, whose basis is the set of paths in Q. In this vector space, we may define a binary operation by extending linearly the composition of paths and setting it to be zero whenever to paths cannot be composed. If Q is finite, this endows $\mathbb{K}Q$ with the structure of a $\mathbb{K}$-algebra, where the multiplicative unit is the sum of all lazy paths. This is said to be the path algebra of Q. Examples of path algebras abound. The ring of polynomials in one variable is the path algebra of the quiver with one vertex and one loop. The free algebra in two (noncommuting) variables is the path algebra of the quiver with one vertex and two loops. The path algebra of the quiver $\mathcal{A}_3$ above is the algebra of upper triangular matrices over $\mathbb{K}$. A representation of Q is a sequence of vector spaces, one per vertex of Q, and subsequent linear maps for each arrow. The study of representations of Q is intimately related to the structure of $\mathbb{K}Q$ and it depends fundamentally on the shape/combinatorics of Q. Crucially, path algebras provide a framework for the study of representations for all finite-dimensional algebras, by considering quotients of path algebras by suitable ideals. Path algebras, their quotients and representations constitute a central topic in algebra that uses techniques from combinatorics, linear algebra, algebraic geometry and category theory. Furthermore, it bridges various research topics from operator algebras in functional analysis to cluster algebras in commutative algebra and Lie theory or coherent sheaves over algebraic varieties. Students following this course should have followed courses in linear algebra and undergraduate abstract algebra (namely an introduction to rings). A basic knowledge of categories and functors is useful but not required.

Aim: At the end of the course, students will be able to:

- Identify basic structural properties of algebras coming from graphs and their representations;

- Understand some links between the combinatorics of a quiver and the representation theory of the corresponding path algebra (and its quotients);
- Learn standard (combinatorial, homological, categorical or geometric) in the representation theory of some classes of algebras;

Examination and grading: The exam will consist on a seminar covering a part of a research paper related to the course.

Course contents:

- Quivers, path algebras, quotients of path algebras and their representations.
- Modules vs. Representations.
- Basic properties of algebras coming from graphs and their representations.
- Some classes of algebras: finite-dimensional algebras; Leavitt path algebras; Jacobian algebras of quivers with potentials; incidence algebras of posets.
- Elements of the representation theory of one of the preceding classes:
  - projective, injective and simple representations;
  - combinatorial and homological properties;
  - categorical equivalences;
  - classification results.

References:

- Abrams, G., Ara, P. and Molina, M.S. , Leavitt Path Algebras, Lecture Notes in Mathematics 2191, Springer (2017).
- Assem, I., Skowronski, A. and Simson, D., Elements of the Representation Theory of Associative Algebras: Techniques of Representation Theory, Cambridge University Press (2006).
- Derksen, H., Weyman, J. and Zelevinsky, A., Quivers with potentials and their representations I: Mutations, Selecta Math. 14 (2008), no. 1, 59–119.
- Rota, G., On the foundations of combinatorial theory. I. Theory of Möbius functions, Z. Wahrscheinlichkeitstheorie und Verw. Gebiete (1964), 340–368.

Intersection Theory

Jakob Scholbach[1]

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Timetable: 16 hrs. Torre Archimede, Room 2BC30

CFU/ECTS: 3

SS: MATH-02/B

Course requirements: Algebraic geometry

Examination and grading: There will be an oral exam at the end of the course.

Aim: The aim of the course is to introduce the students to intersection theory and to give a first idea of the theory of motivic sheaves.

Course contents:

- Chow groups, including higher Chow groups
- K-theory
- Characteristic classes
- The theorem of Grothendieck-Riemann-Roch
- Introduction to motivic sheaves, six functor formalisms

Bibliography:

1. Eisenbud and Harris: "3264 and all that—a second course in algebraic geometry."
2. Fulton: "Intersection theory"
3. Cisinski and Déglise: "Triangulated categories of motives"

Courses of the “Computational Mathematics” area

Volterra Equations and their Markovian Lift(s)

Ofelia Bonesini[1]

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Email: o.bonesini@lse.ac.uk

Timetable: 16 hrs., Room 2BC30, Torre Archimede.

CFU/ECTS: 3

SS: MATH-03/B

Course requirements: Real Analysis and Stochastic Calculus are considered foundational prerequisites, while a basic knowledge of stochastic modelling for finance is helpful yet not necessary

Examination and grading: A 30-minute seminar on one of the selected readings proposed in class

Aim:

1. Understand a generalisation of standard SDEs (SVEs) for which Markovianity and semimartingality do not hold.
2. Analyse Rough Volatility models to understand why fractional Brownian motion is particularly suitable for financial modeling.
3. Explore the infinite dimensional setting established by the use of Markovian lifts, in particular by comparing the different lifting techniques (measure-valued, curve-valued). Apply lifts to restore Markovianity and help in functional Itô calculus.
4. Indentify the connection between SVEs and Path-Dependent PDEs by linking the two to backward stochastic PDEs.
5. Identify the key challenges in the established methods to detect potential further developments and improvements in the established theory and discover new applications.

Course contents:

1. Preliminaries on Stochastic Volterra Equations (SVEs)
   - Definition and main features
   - Comparison with standard SDEs: loss of Markovianity and semimartingality
   - Existence and uniqueness results (in particular, for the singular kernel case)
2. Applications in finance: rough volatility models
   - Fractional Brownian motion and its role as the main player in volatility modelling
   - Rough Bergomi model: definition and its meaning
   - Connections between SVEs and (multifactor) fractional stochastic volatility models
3. Markovian lifts

- Markovian lifts foundation: restoring Markovianity in SVEs
- Measure-valued lift: genesis, how this is used in practice, and limitations
- Curve-valued lift: link with forward variance curves in finance
- Approximation techniques via Markovian representations

4. SVEs and Path-dependent PDEs
   - Connections between SVEs, PPDEs (path-dependent PDEs), and BSPDEs (backward stochastic PDEs)
   - Integral representations and Laplace transform methods
5. Advanced topics and open problems
   - Recent research on uniqueness and weak solutions for SVEs
   - Applications of Itô-type formulas in Volterra processes
   - Open questions in Markovian approximations and numerical methods

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3. E. Abi Jaber and O. El Euch, Markovian structure of the Volterra Heston model, Statistics and Probability Letters, 149 (2019), pp. 63–72.
4. E. Abi Jaber and O. El Euch, Multifactor approximation of rough volatility models, SIAM journal on financial mathematics, 10 (2019), pp. 309–349.
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6. F. E. Benth, N. Detering, and P. Kruehner, Stochastic Volterra integral equations and a class of first-order stochastic partial differential equations, Stochastics, 94 (2022), pp. 1054–1076.
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11. L. Coutin and L. Decreusefond, Stochastic Volterra equations with singular kernels, in Stochastic analysis and mathematical physics, Springer, 2001, pp. 39–50.
12. C. Cuchiero and J. Teichmann, Markovian lifts of positive semidefinite affine Volterra-type processes, Decisions in Economics and Finance, 42 (2019), pp. 407–448.

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15. Y. Hamaguchi, Markovian lifting and asymptotic log-Harnack inequality for stochastic Volterra integral equations. ArXiv preprint, (2023).
16. Y. Hamaguchi, Weak well-posedness of stochastic Volterra equations with completely monotone kernels and non-degenerate noise. ArXiv preprint, (2023).
17. P. Harms and D. Stefanovits, Affine representations of fractional processes with applications in mathematical finance, Stochastic Processes and their Applications, 129 (2019), pp. 1185–1228.
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23. D. J. Prömel and D. Scheffels, Pathwise uniqueness for singular stochastic Volterra equations with Hölder coefficients, Stochastics and Partial Differential Equations: Analysis and Computations, (2024), pp. 1–59.
24. F. Viens and J. Zhang, A martingale approach for fractional Brownian motions and related path dependent PDEs, The Annals of Applied Probability, 29 (2019), pp. 3489–3540.
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26. Z. Wang, Existence and uniqueness of solutions to stochastic Volterra equations with singular kernels and non-Lipschitz coefficients, Statistics and probability letters, 78 (2008), pp. 1062–1071.

Stochastic optimal control

Alekos Cecchin[1]

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Timetable: 16 hrs., expected starting date: October/November 2025, Room 2BC30, Torre Archimede.

CFU/ECTS: 3

SS: MATH-03/B

Course requirements: Basic knowledge of stochastic calculus (Brownian motion, stochastic differential equations, filtrations, martingales, ...), as presented, for example, in the course on stochastic analysis of the mater degree. Some concepts will be recalled during the course.

Examination and grading: Oral presentation of a research paper related to the topics covered in the course, based on student's interest.

Aim: Introduce the classical tools to analyze stochastic optimal control problems, such as dynamic programming, viscosity solutions, backward SDEs, and then use these methods to study either some applications or mean field control problems.

Course contents: Introduction to the classical theory of stochastic control problems with some motivating example from economics and finance. These problems consist in minimizing a cost in which the state variable is given by a controlled stochastic differential equation driven by a Brownian motion. The course will cover some of the following topics:

- Dynamic programming principle: value function, Hamilton-Jacobi-Bellman equation, verification theorem, viscosity solutions of second order fullt nonlinear PDEs;
- Backward stochastic differential equations: representation of the value function for the weak formulation, equivalence of strong and weak formulation, necessary conditions for optimality given by the stochastic Pontryagin's maximum principle, relation with dynamic programming equation;
- Optimal stopping problems, free boundary problems, variational inequalities;
- Mean field control problems, also called optimal control of McKean-Vlasov dynamics. In these problems, the cost and the coefficients of the state equation depend also on the law of the state process, and can be reformulated as optimal control of the Fokker-Planck equation. Hamilton-Jacobi-Bellman equation stated in the Wasserstein space of probability measures.
- Linear-quadratic-gaussian optimal control problems. Applications to economic and financial models.

General enrichment strategies for finite element methods to solve Poisson problem with Dirichlet boundary conditions

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Timetable: 16 hrs.; First lecture on October, 2025, Torre Archimede, Room 2BC30

CFU/ECTS credits: 3

SS: MATH-05/A

Introduction and background requirements:

The finite element method (FEM) is a widely used numerical tool for solving partial differential equations over a domain $\Omega \subset \mathbf{R}^d$, with $d \geq 2$. Its popularity is largely due to its flexibility in handling complex geometries. In FEM, the domain Ω is subdivided into smaller subdomains, over which local approximations are constructed. The global approximation is a piecewise-defined function formed by these local approximations. A finite element is locally defined as a triplet

$$\mathcal{S} = (E, W_k, \Sigma),$$

where:

- E is a polytope;
- W_k is a k-dimensional vector space of functions mapping E to $\mathbf{R}$;
- $\Sigma = \{\mathcal{L}_i \,:\, i = 1, \dots, k\}$ is the set of degrees of freedom, with W_k being Σ-unisolvent, meaning that if $f \in W_k$ and

$$\mathcal{L}_i(f) = 0, \quad i = 1, \dots, k,$$

  then $f = 0$.

The standard triangular linear finite element is commonly used in the applications for its simplicity. It is locally defined as

$$\mathcal{P}_1 = \left(E, \Pi_1(E), \Sigma^{\text{lin}}\right),$$

where

- E is a triangle with vertices $\boldsymbol{v}_1$, $\boldsymbol{v}_2$ and $\boldsymbol{v}_3$;
- $\Pi_1(E)$ is the space of all bivariate polynomials of degree one;
- $\Sigma^{\text{lin}} = \{\mathcal{L}_i(f) = f(\boldsymbol{v}_i) \,:\, i = 1, 2, 3\}$.

However, linear approximations often fail to adequately capture certain complex features of the solution, such as oscillations, exponential decay, or boundary-layer phenomena, particularly in regions with sharp gradients or near boundaries. To address these limitations and improve

FEM accuracy, a common approach involves enriching the finite element space with additional functions specifically designed to capture these complex behaviors. For example, if the solution exhibits sinusoidal characteristics, it is beneficial to enrich the finite element space with sinusoidal functions to better capture these oscillations. Enrichment is achieved by augmenting the finite element $\mathcal{S}$ with *suitable* enrichment functions $e_1, \dots, e_N$ and corresponding functionals

$$\left\{\mathcal{F}_j^{\mathrm{enr}} \,:\, j = 1, \dots, N\right\},$$

forming an enriched triplet

$$\mathcal{S}^{\mathrm{enr}} = \left(E, W_k^{\mathrm{enr}}, \Sigma^{\mathrm{enr}}\right),$$

where

- $W_k^{\mathrm{enr}} = W_k \oplus \{e_1, \dots, e_N\}$
- $\Sigma^{\mathrm{enr}} = \left\{\mathcal{L}_i, \mathcal{F}_j^{\mathrm{enr}} \,:\, j = 1, \dots, N,\, i = 1, \dots, k\right\}.$

In order to enrich the finite element $\mathcal{S}$ the following question must be answered:

- How to properly choose the enrichment functions $e_1, \dots, e_N$, so that the triple $\mathcal{S}^{\mathrm{enr}}$ is a finite element?
- What characteristics should the enrichment functions $e_1, \dots, e_N$ possess for optimal performance?
- How do we select the enrichment functions based on the differential problem at hand?
- What improvements in error bounds can be achieved with this enrichment?

This course is designed for students who possess a basic understanding of numerical analysis and it focuses specifically on enrichment strategies for the Poisson problem with Dirichlet boundary conditions, although these strategies are applicable to a broader class of elliptic boundary value problems. Familiarity with linear algebra is essential, and basic MATLAB programming skills are recommended for the practical implementation of the discussed methods.

Aim:
By the end of the course, students are expected to:

- Understand the fundamental principles of FEM and EFEM.
- Learn to apply FEM and EFEM to solve practical engineering problems.
- Develop the skills necessary to implement enrichment strategies in numerical simulations.

Examination and grading: Solving exercises and giving a seminar on a course-related topic.

Course contents: The course will cover the following topics:

- Introduction to finite element methods.
- Limitations of standard triangular and simplicial linear finite elements.
- Overview of enrichment strategies for finite element methods
- Conforming and nonconforming enrichment approach.
- Enrichment strategies for specific finite elements and error bounds in L^1 and L^∞ norm.
- Implementation of enriched finite element methods to solve the Poisson problem with Dirichlet boundary conditions.

Functional convex ordering of stochastic processes: a constructive approach with applications to Finance

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Timetable: 16 hrs., Room 2BC30, Torre Archimede.

CFU/ECTS: 3

SS: MATH-03/B

Course requirements: Probability Theory, Stochastic processes, stochastic calculus.

Examination and grading: It can be a standard exam with or without documents or more likely the reading of research papers combined with numerical experiments.

Aim:

- Familiarize the audience with the different notions of convex order between random variables and their links with the usual risk measures in finance.
- Extend these notions to a functional framework in order to apply it to Markovian or non-Markovian stochastic processes.
- Analyze the connections between convex order and propagation of convexity by a semi-group associated with various Markov processes.
- Different families of processes will be studied: ARCH processes (discrete time), Brownian or jump diffusion processes, solutions of McKean-Vlasov equations, stochastic Volterra processes.
- Applications to the sensitivity of path-dependent options to ”functional volatility” will be detailed.
- Most of the results will be obtained by passing to the limit from the simulable numerical approximation schemes, of the Euler scheme type, which makes it possible to define effective approximation protocols respecting convex ordering and convexity propagation for the calculation of prices of complex optional products having a path-dependent payoff.

Course contents:

- Convex ordering: definitions and first (static) examples
  - Convex ordering(s) for Rd-valued random vectors
  - Characterization of convex orders
  - First examples: convex ordering of Gaussian vectors, European vanilla options with convex payoff in a Black-Scholes model, Value-at-Risk and Expected shortfall.

    - Toward functional order: the case of Asian option.
- Functional convex ordering(s): definition and characterization
    - Propagation of convexity
    - The case of martingale (and scaled) Brownian diffusions
    - Application to path-dependent European options convex payoffs in local volatility models
    - Extension to jump diffusions (SDEs driven by Lèvy processes)
- From European to American path-dependent options for Brownian and jump diffusions
- Convex ordering for McKean-Vlasov SDEs
- Application to the comparison of mean-field games (optional)
- Convex ordering in a non-Markovian framework: the case of stochastic Volterra equations.
- Application to variance swaps in a Quadratic rough Heston stochastic volatility model.

Bibliography:

P. Carr, C.-O. Ewald and Y.Xiao, On the qualitative effect of volatility and duration on prices of Asian options, Finance Research Letters, 5(3):162–171, 2008.

B. Jourdain, G. Pag‘es, Convex ordering for stochastic Volterra equations and their Euler schemes, Fin. and Stoch., 29(1):1-62, 2025.

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F. Hirsch, B. Roynette, C. Profeta and M. Yor, Peacocks and Associated Martingales, with Explicit Constructions, Springer, 2011.

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Linear Parabolic Equations in Hilbert Spaces: analysis and numerical approximation

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Timetable: 16 hrs.; the course will be held in the second semester.

CFU/ECTS: 3

SS: MATH-05/A

Introduction and background requirements: real and functional analysis, basics of numerical analysis. All the essential notions will be briefly recalled during the lectures.

Aim: The course will offer, in the framework of linear parabolic PDEs, the opportunity of running into the whole scientific process of analyzing a mathematical problem, constructing its solution, and developing a robust numerical approximation method by exploiting the same properties of the problem that surfaced in the analysis step.

Examination and grading: either oral examination on the content of the course, or presentation of a related research paper. Whenener the background of the student includes some programming skills, the presentation of numerical experiments might be included in the exam.

Course contents: The course consists of two closely related and complementary parts: the first 10 hours will be devoted to the construction of the solutions of certain classes of linear parabolic equations formulated as evolution equations in Hilbert spaces. The second part of the course (6 hours) concerns the numerical approximation of such solutions. First we consider the semi-discrete approximation by the Faedo-Galerkin approach, then we exploit the properties of analytic semigroups to define a fully-discrete sequence of approximations by means of the Laplace transform and its numerical inversion.

Due to the time constraint, only the most important (and/or instructive) results will be proven, while many others will be only presented and discussed. Some classical equations (e.g., heat eq., convection-diffusion eq., Sobolev eq., and visco-elastic eq.) will be used as examples both to apply the presented theoretical results and verify their hypothesis, and to test the introduced approximation techniques.

Part 1 (10h - 5 lectures)

1) Quadratic forms and linear operators on Hilbert spaces;
2) Accretive operators, generation of contraction and analytic semigroups, relation with Laplace transform;
3) Solving first-order (in time) linear non-degenerte explicit parabolic equations;

4) More general linear parabolic equations than $u_t = \mathcal{L}u + f$: implicit and second-order equations;
5) Classical examples: heat, convection-diffusion, visco-elastic, and Sobolev equations.

Part 2 (6h - 3 lectures)

6) Galerkin method for elliptic problems, standard error estimates;
7) Semi-discretization by Faedo-Galerkin method, error analysis and convergence;
8) Fully discrete approximation by Laplace transform and quadrature, convergence analysis for sectorial operators.