Codes, graphical models, distributed algorithms **

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Calendario: 20 ore, Martedì ore 16.30 - 18.30 e Giovedì ore 16.30 - 18.30. Prima lezione il 21 aprile 2009. Aula DEI/G (Piano 3, Dipartimento di Ingegneria dell'Informazione, Via Gradenigo 6/a).

Prerequisiti: Basic courses on calculus, linear algebra, probability.

Tipologia di esame: 5 weekly homeworks.

Aim: The goal of this course is twofold. We will first give a quick introduction to channel coding theory with a special emphasis on low density parity check codes and iterative decoding techniques which have been developed in the last 15 years and have allowed to concretely achieve the fundamental Shannon limit. Iterative decoding is an instance of the so-called Belief Propagation (BP) algorithm whose range of possible applications is much wider than just coding. In short, BP is a low-complexity algorithm which allows to compute (in an approximative way) the marginals of a stochastic process defined on a graph. BP can be applied to solve a variety of problems in artificial intelligence, statistical inference, estimation, combinatorics. In the second part of the course we will consider such more general instances of BP algorithm, we will establish some theoretical results and we will investigate some connections with statistical mechanics.

Topics:

1. [6h] Codes for reliable transmission over noisy digital channels. Maximum-a-posteriori (MAP) decoding. Shannon theorem. Some important channels: the binary symmetric channel (BSC), the binary erasure channel (BEC), the Gaussian channel. Complexity of a coding scheme. Linear binary codes, syndromes, minimum distances, weight enumerators, spectra, theoretical bounds. Examples of specific codes and ensembles of codes.
2. [6h] Graphical representations of codes, Tanner graphs. MAP inference for codes represented by tree-like graphs. The Belief Propagation (BP) algorithm. BP for graphs with cycles. Analysis for the case of the binary erasure channel (BEC). Low density parity check (LDPC) codes: regular and irregular ensembles. LDPC over the BEC: performance analysis using the density evolution technique, concentration analysis. The peeling algorithm. Extension to more general transmission channels. Digital fountain codes.
3. [8h] Probabilistic graphical models. The Hammersley-Clifford theorem. Distributed inference problems. BP on tree-like graphs and on graphs with cycles. The exactness of BP on Gaussian Markov fields and on graphs with a single loop. Other applications of BP: combinatorial problems on graphs (matchings, spanning trees, colorings). The connection with statistical mechanics: the Bethe free energy interpretation.

** Corso mutuato dalla *Scuola di Dottorato in "Ingegneria dell'Informazione"*

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