Recent research trends in numerical analysis and applied mathematics

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Calendario: 16 ore svolte il Martedì, Giovedì, Venerdì, dalle ore 9.30 alle ore 11.15, Aula 2BC/60, Torre Archimede, dal 21 aprile 2009.

Prerequisiti: Basic Numerical Analysis.

Tipologia di esame: Tesina finale.

SSD: MAT/08

Obiettivi del corso:
The aim of these lectures is to introduce the students to some recent research subjects in numerical analysis and to provide them the theoretical basis for their understanding. Applications will also be discussed.

Programma del corso:

1. **Orthogonal polynomials**
   (a) Classical orthogonal polynomials: definition and properties
   (b) Quasi-orthogonal polynomials
   (c) Formal orthogonal polynomials: definition and properties
2. **Padé approximation**
   (a) Definition and algebraic properties
   (b) Padé-type approximants
   (c) Connection to formal orthogonal polynomials
   (d) Recursive computation
   (e) Connection to continued fractions
   (f) Some elements of convergence theory
   (g) Applications
3. **Extrapolation methods**
   (a) Sequence transformations and convergence acceleration
   (b) What is an extrapolation method?
   (c) Various extrapolation methods
   (d) Vector sequence transformations
   (e) Applications
      i. Treatment of the Gibbs phenomenon
      ii. Web search
      iii. Estimation of the error for linear systems
      iv. Regularization of linear systems

4. **Tools for linear algebra**
   (a) The singular value decomposition
   (b) The bordering method
   (c) The Schur complement
   (d) The Sherman-Morrison formula
   (e) Sylvester identity
   (f) Preconditioning
5. **Topics on linear systems**
   (a) Preconditioning
   (b) Regularization
6. **Projection methods for linear systems**
   (a) Steepest descent
   (b) The conjugate gradient
   (c) Krylov subspace methods
   (d) The methods of Arnoldi and GMRES
7. **An application: web search**
   (a) The Google matrix
   (b) The PageRank vector
   (c) The power method
   (d) Convergence acceleration
   (e) Extrapolation
8. **Computational aspects of linear control**
   (a) The control problem
   (b) Theoretical issues
   (c) Realization
   (d) Model reduction via Padé approximation and Krylov subspace methods
   (e) State estimation

References:

[1] C. Brezinski, *Padé–Type Approximation and General Orthogonal Polynomials*, ISNM, vol. 50, Birkhäuser–Verlag, Basel, 1980.

[2] C. Brezinski, M. Redivo-Zaglia, *Extrapolation Methods. Theory and Practice*, North–Holland, Amsterdam, 1991.

[3] C. Brezinski, *Biorthogonality and its Applications to Numerical Analysis*, Marcel Dekker, New York, 1992.

[4] C. Brezinski, *Projection Methods for Systems of Equations*, North-Holland, Amsterdam, 1997.

[5] C. Brezinski, *Computational Aspects of Linear Control*, Kluwer, Dordrecht, 2002.