Deformation Quantization

Prof. Gilles Halbout

Calendario: 10 ore
Maggio:
26, 27 e 28 dalle 16.30 alle 18.15 in aula 2BC/30
29 e 30 dalle 11.30 alle 13.15 in aula 2BC/30

Programma del corso:
1st course : I will recall definitions of Poisson manifolds and their deformations. We will study two particular examples: symplectic manifolds and dual of Lie algebras. In the first case, locally the manifold is R^n and we give explicit quantizations. In the second case quantizations can be deduced from Poincare-Birkhoff-Witt theorem. From those examples we will introduce the general theory of deformations of algebras and its related complex: the Hochschild cohomology groups gives classes of obstruction.

2nd course: we will introduce structures "up to homotopies" in order to present formality theorems. For examples the tensor fields and the Hochschild cochains are Lie algebras. But they also can be seen as L_∞ algebras (LIe algebra "up to homotopy"). Formality theorems give existence of Lie algebra morphisms "up to homotopies". We will see that given such morphisms, one can deduce existence and classification of deformations of Poisson manifolds. We will end this course presenting Kontsevich's formality's theorem. All formality theorems will first work locally (when manifolds are R^n).

3rd course: most of this course will be dedicated to a proof of formality's theorem. We will follow Tamarkin's approach: tensor fields and Hochschild cochains are not only Lie algebras but also Gerstenhaber algebras "up to homotopy". We will explain this structure and prove existence of Gerstenhaber algebra morphisms "up to homotopy". We will see the role of Drinfeld's associators and Etingoff-Kazhdan's quantization theorem of Lie bialgebras. We will present a picture of the proof, comprehensive for non specialists. We will see that the constructed morphisms are with coefficient over Q.

4th course: we will show how formality theorems can be globalized for real Poisson manifold. We will used Kontsevich's approached as it was explained in Dolgushev's work. This is baised on Fedosov's resolutions and generalized Weyl bundles. We will present all those objects. We will conclude this course presenting new results by Calaque and Van de Berg

5th course: we will stress the holomorphic and algebraic cases. We will present a survey on different quantization approaches of holomorphic manifolds. We will see how globalization methods can be used in that purpose. We will end this course with open question and stress the problem of quantization of Poisson orbifolds.

Responsabile nel consiglio di dottorato: Andrea D'Agnolo